Discussion on “Towards a quantitative characterization of functional states of the brain: from the non-linear methodology to the global linear description” by J. Wackermann

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Abstract

Wackermann (1999) [Wackermann, J., 1999. Towards a quantitative characterization of functional states of the brain: from the non-linear methodology to the global linear description. Int. J. Psychophysiol. 34, 65–80] proposed Ω–V system for describing the global brain macro-state, in which V complexity was used to quantify the degree of synchrony between spatially distributed EEG processes. In this paper the effect of signal power on V complexity is discussed, which was not considered in Wackermann’s paper (1999). Then an improved method for eliminating the effect of signal power on V complexity is proposed. Finally a case study on the degree of synchrony between two-channel EEG signals over different brain regions during hand motor imagery is given. The results show that the improved Ω complexity measure would characterize the true degree of synchrony among the EEG signals by eliminating the influence of signal power.

Keywords: Ω complexity; The degree of synchrony; Covariance matrix; ERD/ERS; Eigenvalue

1. Introduction

Ω complexity of multichannel EEG has been used in various researches such as sleep and wakefulness, brain maturity, sensory process, etc. (Szelenberger et al., 1996; Wackermann, 1997; Kondakor et al., 1997). It has been proved that Ω complexity could quantify spatial complexity and the degree of synchrony between the distributed spatially multichannel signals (Wackermann, 1996, 1999; Joydeep, 2000). Large value of Ω complexity indicates low synchrony of EEG signals over the different electrodes. The smaller the value, the higher synchrony was (Wackermann, 1996, 1999; Pizzagalli et al., 2000). The high synchrony of multichannel EEG signals reveals the high correlation of signals. Thus Ω complexity provides us with a good index for describing

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correlation among multichannel signals, so that the classical two-channel estimator (such as coherence, covariance) for quantifying correlation between two signals can be well generalized. However, this must be based on the fact that \( \Omega \) complexity is only dependent on the correlation among EEG signals. Neglecting this point, the wrong analysis results may arise, which will be demonstrated in the following sections.

Firstly, let us review the definition of \( \Omega \) complexity proposed by Wackermann (1999). Considering \( N \) EEG samples in the observed time window from \( K \) electrodes to form the voltage vectors \( \{ u_1, \ldots, u_N \} \), where each \( u_i \) \((i=1, \ldots, N)\) is a \( K \)-dimensional state vector representing the spatial distribution of EEG voltages over the \( K \) electrodes at the \( i \)th sample. Before calculating \( \Omega \) complexity, the data are assumed to have been already centered to zero mean value in all channels and transformed to the average reference. Then \( \Omega \) complexity of \( K \)-channel EEG can be calculated as follows.

\[
C = \frac{1}{N} \sum_{n=1}^{N} u_n u_n^T
\]

(1)

\[
\log \Omega = - \sum_{i=1}^{K} \lambda_i' \log \lambda_i'
\]

(2)

where \( C \) is a \( K \times K \) covariance matrix; \( \lambda_i' = \lambda_i / \sum_{j=1}^{K} \lambda_j \), \( \lambda_i \) is the eigenvalues of the matrix \( C \); \( \lambda_i' \) is the normalized eigenvalues. Here, \( K \)-dimensional voltage vectors constructed from the simultaneous EEG measurements over \( K \) electrodes could be viewed as the EEG trajectories in the \( K \)-dimensional state space. \( \Omega \) complexity assesses the spatial complexity, i.e. the shape of trajectory (‘data cloud’) in the state space. The method decomposes the multichannel EEG data into spatial principal components and then the spatial complexity can be quantified by the extension of ‘data cloud’ along the principal axes. The eigenvalues are not only related to the diagonal elements of matrix \( C \) but also to the off-diagonal elements of \( C \). The diagonal elements correspond to the mean power of each channel signal. The effect of signal power should be considered if \( \Omega \) complexity is used to describe the degree of synchrony between multichannel EEG signals. \( \Omega \) complexity is not independent from the signal power although the eigenvalues are normalized before entering Eq. (2). In the following discussion, it will be demonstrated that the normalized eigenvalues just play a role to limit the range of eigenvalues within 0–1 and the signal power independent of \( \Omega \) should rely on its independence of the original eigenvalues. So only by eliminating effect of the signal power on eigenvalues could \( \Omega \) complexity quantify the true synchrony between multichannel EEG signals.

2. Impact of signal power on the \( \Omega \) complexity

Consider \( N \) EEG samples recorded over \( K \) electrodes in the observed time window, \( X_1, \ldots, X_K \), where \( X_i = (x_{11}, x_{12}, \ldots, x_{1N}) \), \( X_K = (x_{K1}, x_{K2}, \ldots, x_{KN}) \) and they have been centered to zero mean in time and spatial domain. Then the covariance matrix can be constructed as:

\[
C_K = \frac{1}{N} \begin{pmatrix}
\sum x_{11}^2 + x_{12}^2 + \cdots + x_{1N}^2 \\
\sum x_{11} x_{12} + x_{12} x_{22} + \cdots + x_{1N} x_{KN} \\
\vdots \\
\sum x_{K1} x_{K2} + x_{K2} x_{K2} + \cdots + x_{KN} x_{KN}
\end{pmatrix}
\]

(3)

where \( P_1 = (x_{11}^2 + x_{12}^2 + \cdots + x_{1N}^2)/N \) and \( P_K = (x_{K1}^2 + x_{K2}^2 + \cdots + x_{KN}^2)/N \) reflect the mean power of EEG in the 1st channel and the \( K \)th channel, respectively; \( \text{Cov} C_{1,K} = \text{Cov} C_{K,1} = (x_{11} x_{K1} + x_{12} x_{K2} + \cdots + x_{1N} x_{KN})/N \) reflects the mean covariance between two signals of the 1st and \( K \)th channel. The covariance matrix is real symmetric.

According to Gerschgorin’s Circle theorem, all the eigenvalues of matrix \( C_K \) are distributed within the combining circles centered with the diagonal elements \( P_1, \ldots, P_K \). The radius of each circle is the sum of absolute values of all off-diagonal elements in the corresponding row. The eigenvalues depend not only on \( P_1, \ldots, P_K \), i.e. the mean power of each channel
signal, but also on the off-diagonal elements of the covariance matrix.

To simplify the problem, let us consider the two-channel $\Omega$ complexity (short for TCOC) as an example. The corresponding covariance matrix is:

$$C_2 = \begin{pmatrix} P_1 & Cov_{1,2} \\ Cov_{2,1} & P_2 \end{pmatrix}$$  \hspace{1cm} (4)

For the real symmetric matrix $C_2$, two eigenvalues are located within the real number range $P_1 \pm Cov_{1,2}$ and $P_2 \pm Cov_{1,2}$, respectively, if $|Cov_{1,2}| < (P_1 + P_2)/2$ is satisfied. Otherwise, two eigenvalues are in the range of combining $P_1 \pm Cov_{1,2}$ with $P_2 \pm Cov_{1,2}$. With $|E - C_2| = 0$, the exact eigenvalues $\lambda$ can be calculated:

$$\lambda_{1,2} = \frac{P_1 + P_2}{2} \pm \sqrt{(\frac{P_1 + P_2}{2})^2 + (Cov_{1,2})^2}$$  \hspace{1cm} (5)

Then the normalized eigenvalues $\lambda'$ can be obtained:

$$\lambda'_{1,2} = \frac{1}{2} \pm \frac{1}{2(P_1 + P_2)} \times \sqrt{(P_1 - P_2)^2 + 4(Cov_{1,2})^2}$$  \hspace{1cm} (6)

From Eq. (6), it can be seen that after normalization, the range of eigenvalues are limited within 0–1. However the normalized eigenvalues $\lambda'_{1,2}$ are still related to $P_1$, $P_2$, i.e. the mean power of two channel signals. Saying that $\Omega$ complexity is independent of total power by normalized eigenvalues (Wackermann, 1999) is not reasonable. Since log $\Omega$ corresponds to the ‘entropy’ of the spectrum of eigenvalues of the covariance matrix (Szelenberger et al., 1996), the value of TCOC is actually related to the difference between the two eigenvalues. Eq. (6) shows that TCOC is determined not only by Cov$_{1,2}$ but also by $(P_1 - P_2)$ and $(P_1 + P_2)$. Cov$_{1,2}$ describes the degree of synchrony between the two signals while $(P_1 - P_2)$ and $(P_1 + P_2)$ contribute nothing to synchrony. Thus the effect of the signal power on the TCOC should be removed if TCOC is used to quantify the synchrony between two signals. Similarly, the multichannel $\Omega$ complexity could not characterize the true degree of synchrony between the signals without considering the effect of signal power.

### 3. Eliminating effect of signal power on $\Omega$ complexity

To eliminate the influence of signal power and reveal the true synchrony between the signals, it is necessary to preprocess EEG signal before calculating the eigenvalues. Since the mean power of each channel signal has effect on the eigenvalues, the simplest method to eliminate its influence is to normalize EEG of each channel by the corresponding mean power before constructing covariance matrix. Then the diagonal elements of the resultant covariance matrix are equal to 1 while the correlation between EEG signals still keeps unchanged.

For the TCOC, the normalized covariance matrix is as follows:

$$C_{2\text{nor}} = \begin{pmatrix} 1 & Cov_{1,2}/\sqrt{P_1P_2} \\ Cov_{2,1}/\sqrt{P_1P_2} & 1 \end{pmatrix}$$  \hspace{1cm} (7)

From Eq. (7), it can be seen that after the normalization by mean power, the eigenvalues of $C_{2\text{nor}}$ are only associated with the off-diagonal element $Cov_{1,2}/\sqrt{P_1P_2}$ which just corresponds to the normalized covariance. Covariance normalized by the product of individual variance is called the correlation function coefficient, which has been suggested to provide a means for studying the correlation between the two signals (Nunez et al., 1997; Bendat and Piersol, 1986). Similar to the correlation function, the normalized TCOC can be used to characterize the true spatial synchrony and coherence of the two distributed EEG signals.

For the $K$-channel $\Omega$ complexity, the normalized covariance matrix is as follows:

$$C_{K\text{nor}} = \begin{pmatrix} 1 & \ldots & Cov_{1,K}/\sqrt{P_1P_K} \\ \vdots & \ddots & \vdots \\ Cov_{K,1}/\sqrt{P_1P_K} & \ldots & 1 \end{pmatrix}$$  \hspace{1cm} (8)

All the eigenvalues only depend on the correlation function coefficients between all the signal pairs of $K$ channels. $K$-channel $\Omega$ complexity as a function of the eigenvalues of $C_{K\text{nor}}$ can quantify the true spatial complexity and the degree of synchrony among EEG signals distributed over $K$ channels. Therefore, an improved multichannel $\Omega$ complexity measure could
be obtained by eliminating the effect of signal mean power.

4. A case study

To demonstrate the effect of signal power on $\Omega$ complexity, a case study is given. Here the used train datasets are from BCI2003 competition website provided by Graz University of technology. The datasets include 140 labeled trials of imaginary hand movements, with an equal number of left and right hand trials. Three bipolar EEG channels were measured over the anterior and posterior of C3, Cz and C4 with inter-electrode intervals of 2.5 cm. In each of 9-s trial, the subject was asked to imagine left or right hand movement by the arrow direction on the screen after 3-s relaxation. The detailed descriptions can be found in relevant website and references (Schlogl et al., 1997; Neuper et al., 1999).

Studies at Graz University of Technology have shown that the unilateral hand motor imagery would result in the event-related desynchronization/synchronization (ERD/ERS) of alpha and beta rhythmic activities over the contralateral and ipsilateral hand area, respectively (Pfurtscheller and Lopes da Silva, 1999). TCOC of EEG within 8–30 Hz from the contralateral and mid-central region (close to Cz), from the ipsilateral and the mid-central region were investigated, respectively. The purpose of this paper is to show the effect of signal power on the TCOC, so the study on the synchrony between EEG within subdivisible frequency band is not discussed.

Generally before calculating TCOC, EEG data need to be centered to zero mean and transformed to average reference. For the used dataset, there is no influence from reference and no need to transform data to average reference since EEG was recorded by bipolar derivation acting as a spatial high-pass filter to allow local cortical activity to be measured (Pfurtscheller et al., 1997; Nunez et al., 1997). To obtain TCOC time course, 1-s data segment is extracted from the trial to calculate two-channel $\Omega$ complexity. By shifting the data segment sample by sample from the start to the end of the trial, and calculating averaged $\Omega$ across all the trials for each segment, TCOC time sequence can be obtained. With ensemble of all the trials, the information of revealing short time changes in TCOC due to the hand motor imagery could be yielded (Andrew and Pfurtscheller, 1996).

The normalized and non-normalized TCOC time course for the left or right hand motor imagery are shown in Fig. 1(a,b) and (c,d), respectively. As a comparison, ERD time course within 8–30 Hz quantified by the classical power method is also given in Fig. 1(e,f), where 0.5–1.5 s was defined as the reference interval and ERD time course was computed as the percentage changes related to this reference interval (Pfurtscheller and Lopes da Silva, 1999; Kalcher and Pfurtscheller, 1995). TCOC at time $t$ is calculated by a 1-s segment EEG preceding $t$, so both the normalized and non-normalized TCOC time course delayed ERD time course by 1 s.

Compared with the normalized TCOC time course, the non-normalized TCOC time course shows some contradictory behavior, which suggests that the normalization by signal power will impact the TCOC time course. From Fig. 1(e,f), it can be seen that during left- or right-hand motor imagery, alpha and beta rhythmic activities over both the contralateral and mid-central regions are desynchronized. The desynchronized rhythms indicate that each of the underlying areas becomes active (Andrew and Pfurtscheller, 1996), so the degree of synchrony between these rhythms will decrease. The single-channel complexity analysis of the event-related EEG data shows that ERD/ERS corresponds to the increase and decrease of EEG complexity, respectively (Pei et al., 2004). $\Omega$ complexity characterizes the spatial complexity and the degree of synchrony between the distributed spatially EEG signals. It is reasonable to expect that during the unilateral hand motor imagery, TCOC of EEG from contralateral and mid-central regions would increase and TCOC of EEG from ipsilateral and mid-central regions would decrease. In Fig. 1(a,b), the normalized TCOC time course is consistent with the expected descriptions above and the reported findings by Andrew and Pfurtscheller (1996). However, in Fig. 1(c,d), the non-normalized TCOC time course is contradictory against the findings reported (Andrew and Pfurtscheller, 1996) and the true correlation between the two desynchronized EEG activities cannot be reflected. Combining the analysis of ERD/ERS with the algorithm of non-normalized $\Omega$ complexity, it can be easily demonstrated that the non-normalized TCOC time course
was influenced by the signal power. The effects of signal power could be considered from the two aspects. On the one hand, the signal power corresponding to ERD/ERS at the onset of imagination display the pronounced decrease and increase, respectively, relative to that in the reference period. The non-
normalized TCOC calculated by Eq. (6) reflects the more changes of signal power than the synchrony of two EEG signals in different time periods. Thus the true synchrony of two EEG signals changing with time cannot be quantified by the non-normalized TCOC. On the other hand, at the onset of imagination, the total power of EEG from ipsilateral and mid-central region is larger than that from contralateral and mid-central region. Thus the large differences of signal power from two brain regions have great effects on the comparison of the non-normalized TCOC between the two brain regions.

In summary, without considering the effect of signal power on $\Omega$ complexity the wrong results may arise.

5. Discussions

The normalized TCOC is obtained by the eigenvalues of the normalized correlation matrix $C_{2\text{nor}}$. By $|\lambda E - C_{2\text{nor}}|=0$ and Eq. (7), the two eigenvalues can be calculated as follows:

$$\lambda_{1,2} = 1 \pm \frac{\text{Cov}C_{1,2}}{\sqrt{P_1 P_2}}$$

The normalized TCOC could be expressed as a function of the two eigenvalues and further as a function of the correlation coefficient. Thus the normalized TCOC is actually equivalent to correlation function coefficient (Nunez et al., 1997; Bendat and Piersol, 1986) which has been proposed to describe the coherence between two signals. The larger value of the correlation coefficient is, the lower the normalized TCOC is, which corresponds to the high synchrony of two signals. The difference between the correlation coefficient and the normalized TCOC is that the value of the former is within 0–1 but the value of the latter is within 1–2. The normalized multichannel $\Omega$ complexity could be regarded as an extension of correlation function coefficient, which provides with us a good method for describing correlation of more than two signals. Correlation function coefficient is the normalized covariance (Nunez et al., 1997; Bendat and Piersol, 1986). Similarly, it is necessary to remove the effect of signal power for multichannel $\Omega$ complexity characterizing correlation or synchrony of more than two signals.

The normalized eigenvalues could not result in the total power independent of $\Omega$ complexity (Wackermann, 1999). Each eigenvalue is limited within the same range 0–1 after normalization by sum of all the eigenvalues. Then the value of $K$-channel $\Omega$ complexity is limited within $1-K$ so that it is possible to make comparison of different brain states under the uniform range. The total power independence of $\Omega$ complexity should rely on its independence of the original eigenvalues. So after eliminating the effect of signal power on eigenvalues, the improved $\Omega$ complexity would provide a measure for quantifying the true degree of synchrony between the distributed spatially EEG signals.

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References


